In the name of God.

Homework-4 This set of exercises is due by 4/1/97.

E1: For any function $f : A \to B$ the following conditions are equivalent:

i) f is a bijection.

ii) f^{-1} is a function mapping B into A.

iii) There is a function $g : B \to A$ such that $f \circ g = Id_B$ and $g \circ f = Id_A$.

E2: Suppose that A is a set, and f maps $\mathscr{P}(A)$ into $\mathscr{P}(A)$. Assume that

$$X \cap f(Y) \neq \emptyset \longleftrightarrow f(X) \cap Y \neq \emptyset$$

for all X, Y in $\mathscr{P}(A)$. Show that if $\{B_i\}_{i \in I}$ is a family of subsets of A, then:

$$f\left(\bigcup_{i\in I} B_i\right) = \bigcup_{i \in I} f(B_i)$$

E3: Prove the following equality:

$$\prod_{i\in I}(\bigcap_{j\in J_i} A_{i,j}) = \bigcap_{f \in \prod\limits_{i\in I} J_i} (\prod_{i\in I} A_{i,f(i)})$$

E4: Suppose that $f : A \to B$. Show that the following conditions are equivalent:

i) f is onto.

ii) For all C, g, h, if $g : B \to C$, $h : B \to C$, and $g \circ f = h \circ f$, then $g = h$.

E5: Suppose that $F : \mathscr{P}(A) \to \mathscr{P}(A)$ is such that:

$$X \subseteq Y \subseteq A \Rightarrow F(X) \subseteq F(Y)$$

And let:

$$B = \cap\{X \subseteq A : F(X) \subseteq X\}$$
$$C = \cup\{X \subseteq A : F(X) \subseteq X\}$$

Prove that:

i) $F(B) = B, F(C) = C$.

ii) If $F(X) = X$, then $B \subseteq X \subseteq C$.