سری دوم تمارین مبانی نظریه محاسبه

1. Suppose L is a subset of $\{a, b\}^*$. If $x_0, x_1, \ldots$ is a sequence of distinct strings in $\{a, b\}^*$ such that for every $n \geq 0$, x_n and x_{n+1} are L-distinguishable, does it follow that the strings $x_0, x_1, \ldots$ are pairwise L-distinguishable? Either give a proof that it does follow, or find an example of a language L and strings $x_0, x_1, \ldots$ that represent a counterexample.

2. For each of the following languages $L \subseteq \{a, b\}^*$, show that the elements of the infinite set $\{a^n \mid n \geq 0\}$ are pairwise L-distinguishable.

I. $L = \{a^i b^j \mid j = i \text{ or } j = 2i\}$

II. $L = \{x \in \{a, b\}^* \mid \text{no prefix of } x \text{ has more b's than a's}\}$

III. $L = \{ww \mid w \in \{a, b\}^*\}$

3. Describe decision algorithms to answer each of the following questions.

a. Given two FAs M_1 and M_2, are there any strings that are accepted by neither?

b. Given an FA $M = (Q, \Sigma, q_0, A, \delta)$ and a state $q \in Q$, is there an x with $|x| > 0$ such that $\delta^*(q, x) = q$?

c. Given an FA M accepting a language L, and given two strings x and y, are x and y distinguishable with respect to L?

d. Given an FA M accepting a language L, and a string x, is x a prefix of an element of L?

e. Given an FA M accepting a language L, and a string x, is x a suffix of an element of L?

f. Given an FA M accepting a language L, and a string x, is x a substring of an element of L?

g. Given two FAs M_1 and M_2, is $L(M_1)$ a subset of $L(M_2)$?

h. Given two FAs M_1 and M_2, is every element of $L(M_1)$ a prefix of an element of $L(M_2)$?

4. Show that for every $n>0$ a FA like M_n exist such that the language that M_n accepts is.
$L = \{w \in \{0,1\}^* \mid w \text{ is a number in base 2 and } w \overset{n}{\equiv} 0\}$